Induced charge matching and Wess–Zumino term on quantum modified moduli space

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Recently it was proposed that matching of global charges induced in vacuum by slowly varying, topologically non-trivial scalar fields provides consistency conditions analogous to the 't Hooft anomaly matching conditions. We study matching of induced charges in supersymmetric $SU(N)$ gauge theories with quantum modified moduli space. We find that the Wess–Zumino term should be present in the low energy theory in order that these consistency conditions are satisfied. We calculate the lowest homotopy groups of the quantum moduli space, and show that there are no obstructions to the existence of the Wess–Zumino term at arbitrary N. The explicit expression for this term is given. It is shown that neither vortices nor topological solitons exist in the model. The case of softly broken supersymmetry is considered as well. We find that the possibility of global baryon number violating vacuum is strongly disfavored.

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I. INTRODUCTION

Low-energy dynamics of gauge theories strongly coupled in the infrared often can be described in terms of effective theories. For instance, the soft limit of quantum chromodynamics (QCD) is the theory of self-interacting pion fields. In more sophisticated models (e.g., supersymmetric gauge theories), the effective low-energy theories exhibit interesting content of composite fermions as well. This property may be important for the construction of composite models of quarks and leptons.

Most of the results concerning the infrared behavior of supersymmetric gauge theories are obtained without detailed calculations which are problematic in the strong coupling domain. Instead, one makes use of general properties, such as symmetries and holomorphy. One of the most powerful tools for constraining the low-energy spectra is provided by the 't Hooft anomaly matching conditions [1]. Namely, one introduces topologically non-trivial background gauge fields corresponding to the flavor symmetry group and checks that the anomalies in global currents are the same in the microscopic and effective theories. The basis for the anomaly matching conditions is provided by the Adler-Bardeen non-renormalization theorem [2].

Recently, it was suggested to consider topologically non-trivial, slowly varying in space scalar fields as other probes of effective theories [3]. The corresponding matching conditions emerge due to the fact that such background scalar fields generically induce global charges in vacuum. These charges are the quantities which should match in the microscopic and low energy descriptions of the model. The non-renormalization theorem justifying these matching conditions was proven in Ref. [4].

Induced charge matching in supersymmetric QCD (SQCD) with $SU(N_c)$ gauge group and the number of flavors N_f larger than the number of colors has been discussed in Refs. [3,4]. Models with softly broken supersymmetry were also considered there and it was found that in the latter case the induced charge matching conditions provide new information in addition to the 't Hooft conditions. The present paper is devoted to the study of induced charge matching in SQCD at $N_f = N_c$ when the quantum deformation of the moduli space takes place [5]. It is shown that all constraints are satisfied provided the term analogous to the Wess–Zumino term in QCD is added. This term has been already constructed in the case of $SU(2)$ gauge group in Ref. [6]. Here we extend this construction to the case of $SU(N_c)$ group with arbitrary N_c. In the case of softly broken supersymmetry it is found that induced charge matching conditions require that baryon symmetry is unbroken in the global vacuum.

The paper is organized as follows. In section II we recapitulate the notion of induced charge matching. We consider the QCD case and discuss the role of the Wess–Zumino term there. In section III we study induced charge matching in SQCD with quantum modified moduli space, analyze its topology and construct the analogue of the Wess–Zumino term in this model. The case of softly broken supersymmetry is considered in section IV. In section V we present our conclusions. Appendix A is devoted to the details of the topological analysis of the quantum moduli space and in Appendix B we show that the Wess–Zumino term of SQCD is unambiguous in quantum theory.

II. INDUCED CHARGE MATCHING

To begin with, let us consider QCD with N_c colors and N_f massless fermion flavors. Let ψ^a and $\tilde{\psi}_{\tilde{a}}$, $a, \tilde{a} = 1, \ldots, N_f$, denote left-handed quarks and anti-quarks, respectively. This theory exhibits the global $SU(N_f)_L \times$

$SU(N_f)_R$ symmetry and non-anomalous baryon symmetry, $\psi \to e^{i\alpha}\psi$, $\tilde{\psi} \to e^{-i\alpha}\tilde{\psi}$. To probe the theory, one introduces time-independent background scalar fields $m_b^{\tilde{a}}(\mathbf{x})$ of the following form,

$$m_b^{\tilde{a}}(\mathbf{x}) = m_0 U_b^{\tilde{a}}(\mathbf{x}) \;, \tag{1}$$

where m_0 is a constant and $U_b^{\tilde{a}}(\mathbf{x})$ is an $SU(N_f)$ matrix at each point $\mathbf{x}$.

Let these fields interact with quarks and anti-quarks,

$$\mathcal{L}_{\text{int}} = \tilde{\psi}_{\tilde{a}} m_b^{\tilde{a}} \psi^b + \text{h.c.} \tag{2}$$

In what follows we restrict the form of the background fields by requiring that $U(\mathbf{x})$ tends to a constant at spatial infinity. One can always make this constant equal to unity, $U(\mathbf{x}) \to \mathbf{1}$ as $|\mathbf{x}| \to \infty$, by making use of a global $SU(N_f)$ rotation.

The baryonic current is conserved and obtains non-vanishing vacuum expectation value in the presence of the background scalar fields. To the leading order in momenta, the one-loop result for the induced current is [7]

$$\langle j_B^\mu \rangle = \frac{N_c}{24\pi^2} \epsilon^{\mu\nu\lambda\rho} \,\text{Tr}\left(U\partial_\nu U^\dagger U \partial_\lambda U^\dagger U \partial_\rho U^\dagger \right) \;. \tag{3}$$

It follows from this expression that the baryonic charge induced in vacuum is proportional to the topological number of the background,

$$\langle B \rangle = N_c N[U] \;, \tag{4}$$

where

$$N[U] = \frac{1}{24\pi^2} \int d^3x \; \epsilon^{ijk} \,\text{Tr}\left(U\partial_i U^\dagger U \partial_j U^\dagger U \partial_k U^\dagger \right) \;. \tag{5}$$

This topological property is the basis of the theorem proven in Ref. [4] that states that Eq. (4) does not get renormalized in the full quantum theory provided the expansion in momenta of the background fields is valid. It is worth noting also, that higher derivative terms omitted in Eq. (3) do not contribute to $\langle B \rangle$.

More generally, one does not necessarily introduce masses to all quarks, and considers instead background fields interacting only with some of the flavors. The only requirement is that all fermions coupled to the background fields become massive due to this interaction. Instead of the baryon number one may study generators of other non-anomalous global symmetries which remain unbroken in the presence of the background fields.

The non-renormalization theorem implies that the induced charges should match in the microscopic and effective theories. These matching conditions are analogous, but generally inequivalent, to the 't Hooft conditions.

Let us see how induced charges match in QCD with $N_f \geq 3$. Following Ref. [3] let us consider a general case when a mass term (2) is added to the first N_0 quark flavors. There are two independent generators of the original $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ symmetry group that remain unbroken in the presence of the background fields and act non-trivially on the first N_0 quark flavors. The first one is a generator of a baryon symmetry and the second one transforms first N_0 flavors in the same way as baryon symmetry and acts trivially on the other quarks.

As all fundamental fermions that couple to the background scalar field acquire masses due to this interaction, the derivative expansion is justified. Hence, for slowly varying $m(\mathbf{x})$ one has

$$\langle Q_i \rangle = N_c N[U] \tag{6}$$

for the induced charges $\langle Q_i \rangle$ corresponding to the above two generators.

The low energy dynamics of QCD is described in terms of $SU(N_f)$-valued sigma-model field $V(x)$. The interaction with the background field $m(\mathbf{x})$ induces a potential term into the low-energy effective Lagrangian,

$$\Delta\mathcal{L}_{\text{eff}} = \text{Tr}\left(m^\dagger V + V^\dagger m \right) \;.$$

For slowly varying $m(\mathbf{x})$, the effective potential is minimized at

$$V(\mathbf{x}) = \begin{pmatrix} U(\mathbf{x}) & 0 \\ 0 & \mathbf{1} \end{pmatrix} \;. \tag{7}$$

As we will see shortly, the induced charge matching conditions are satisfied provided the Wess–Zumino term [8] is added in the effective action. This term cannot be written as a four-dimensional integral of a $SU(N_f)_L \times SU(N_f)_R$-invariant non-singular function and is defined as follows [9]. A field configuration $V(x)$ of finite energy defines a map $S^4 \to SU(N_f)$ from compactified spacetime to the space of vacua (moduli space). Since $\pi_4(SU(N_f)) = 0$, the image of this map is a boundary of a five-dimensional submanifold Σ_5 in $SU(N_f)$. The Wess–Zumino term is the integral over this surface,

$$\begin{aligned} \Gamma_{QCD} = \frac{-iN_c}{240\pi^2} \int_{\Sigma_5} d\Omega \; \epsilon^{\mu\nu\lambda\rho\sigma} \\ \times \text{Tr}\left(V^{-1}\partial_\mu V V^{-1}\partial_\nu V V^{-1}\partial_\lambda V V^{-1}\partial_\rho V V^{-1}\partial_\sigma V \right) \;. \end{aligned} \tag{8}$$

This expression is invariant under small deformations of Σ_5 because the integrand is a closed five-form. The discrete ambiguity in the definition of Γ_{QCD} is related to non-zero homotopy group $\pi_5(SU(N_f))$. However, this ambiguity is irrelevant in quantum theory, because $e^{i\Gamma_{QCD}}$ is well-defined. The contribution of the Wess–Zumino term to all global currents except for baryonic one can be obtained by making use of the conventional Noether procedure and reads [9]

$$\begin{aligned} j^{\mu a} = \frac{1}{48\pi^2} \epsilon^{\mu\nu\lambda\rho} \,\text{Tr}\big(T_L^a V^{-1}\partial_\nu V V^{-1}\partial_\lambda V V^{-1}\partial_\rho V \\ + T_R^a \partial_\nu V V^{-1}\partial_\lambda V V^{-1}\partial_\rho V V^{-1} \big) \;, \end{aligned} \tag{9}$$

where T_L^a and T_R^a are $SU(N_f)_L$ and $SU(N_f)_R$ components of a generator of a symmetry transformation. Comparing Eq. (9) with the formulae (6) and (5) for the induced charges and taking into account the vacuum value

(7) of the field $V(x)$, one checks that induced charges match for the unbroken generator of the flavor group $SU(N_f)_L \times SU(N_f)_R$. The expression for the induced baryonic current can be obtained by the formal substitution $T^a_{L,R} \to \mathbf{1}$ in Eq. (9), see Ref. [9] for details. Again, due to Eq. (7) induced baryonic charges match in the fundamental and effective theories. We conclude that induced charges match in QCD due to the presence of the Wess–Zumino term.

III. WESS–ZUMINO TERM IN SQCD

Let us turn now to the case of SQCD with $N_f = N_c = N$. This theory exhibits quantum deformation of the moduli space [5]. Namely, the space of vacua of the microscopic theory is described by the set of holomorphic gauge invariants constructed out of quarks Q^a and antiquarks $\tilde{Q}_{\tilde{b}}$. These invariants are mesons,

$$M^a_{\tilde{b}} = Q^a \tilde{Q}_{\tilde{b}}$$

and (anti)-baryons

$$B = \epsilon_{a_1..a_N} Q^{a_1}..Q^{a_N} \ ,$$

$$\tilde{B} = \epsilon^{\tilde{b}_1..\tilde{b}_N} \tilde{Q}_{\tilde{b}_1}..\tilde{Q}_{\tilde{b}_N}$$

subject to the constraint

$$\det M - B\tilde{B} = \Lambda^{2N} \ , \qquad (10)$$

where Λ is the infrared scale of the theory. The r.h.s of Eq. (10) is of purely quantum origin and indicates the difference between the topologies of the quantum and classical spaces of vacua.

In the low-energy theory of mesons and (anti-)baryons the constraint (10) can be presented as an effective superpotential

$$W = \mathcal{A}(\det M - B\tilde{B} - \Lambda^{2N}) \ , \qquad (11)$$

where $\mathcal{A}$ is the Lagrange multiplier superfield.

Let us probe this theory by adding the scalar background field $m^{\tilde{q}}_p(\mathbf{x})$ with the same properties as above, i.e., by introducing the term

$$m^{\tilde{b}}_a(\mathbf{x}) \tilde{Q}_{\tilde{b}} Q^a \qquad (12)$$

into the superpotential of the fundamental theory. We add the mass terms to *all* quark flavors to avoid the run-away vacuum. Then for the general matrix $U^{\tilde{b}}_a(\mathbf{x})$, the external fields are neutral only under the baryon symmetry[1]. The calculation of the induced baryonic charge in the fundamental theory proceeds as in sect. 2, and we again obtain

$$\langle B \rangle = N_c N[U] \ . \qquad (13)$$

We now turn to effective low energy theory. For slowly varying $m(\mathbf{x})$ the interaction (12) translates into the additional term $\mathrm{Tr}\, mM$ in the effective superpotential (11). As a result, the ground state is described by the following $\mathbf{x}$-dependent expectation values[2] of mesons and baryons,

$$\begin{aligned} M(\mathbf{x}) &= \Lambda^2 U^{-1}(\mathbf{x}) \ , \\ B &= \tilde{B} = 0 \ . \end{aligned} \qquad (14)$$

One can explicitly check that no baryonic charge is induced due to superpotential interaction in the effective theory, as opposed to the supersymmetric models considered in Refs. [3,4]. So, the Wess–Zumino term is needed in complete analogy to the QCD case.

The Wess–Zumino term has been already constructed in the case of $SU(2)$ group [6]. In that theory one combines mesons and baryons into a single 4×4 antisymmetric matrix,

$$V = \left(\begin{array}{cc|c} 0 & B & M \\ -B & 0 & \\ \hline -M^T & & \begin{array}{cc} 0 & \tilde{B} \\ -\tilde{B} & 0 \end{array} \end{array} \right) \ . \qquad (15)$$

In terms of this matrix, the Wess–Zumino term can be written as follows[3],

$$\begin{aligned} \Gamma_{SU(2)} = \frac{1}{240\pi^2} \mathrm{Im} \int_{\Sigma_5} d\Omega \ \epsilon^{\mu\nu\lambda\rho\sigma} \qquad (16) \\ \times \mathrm{Tr}\left(V^{-1}\partial_\mu V V^{-1}\partial_\nu V V^{-1}\partial_\lambda V V^{-1}\partial_\rho V V^{-1}\partial_\sigma V \right) \ . \end{aligned}$$

In fact, this equation defines only the bosonic part of the Wess–Zumino action. However, it is sufficient for our purposes, since it is this part that is relevant for charge matching. The Wess–Zumino term (16) is written in the holomorphic form, see Ref. [10] for a discussion of why this is possible.

As pointed out in Ref. [6], the generalization of Eq. (16) to the case of $SP(N)$ group is straightforward. So, let us consider $SU(N)$ gauge group at $N \geq 3$.

In order that the Wess–Zumino term could be constructed, the fourth homotopy group of the quantum moduli space $\mathcal{Q}$ determined by Eq. (10) should be trivial, $\pi_4(\mathcal{Q}) = 0$. Moreover, if $\pi_5(\mathcal{Q})$ is non-trivial, the value of the Wess–Zumino functional Γ on its generators should be equal to $2\pi n$, $n \in \mathbb{Z}$, so that $e^{i\Gamma}$ is well-defined in quantum theory.

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[1] In section IV we discuss a special case where additional unbroken global symmetries are present.

[2] Hereafter we use the same notations for superfields and their scalar components.

[3] Note that the matrix V is non-degenerate due to the constraint (10).

In Appendix A it is shown that

$$\mathcal{Q} \sim \Sigma \left(\Sigma \left(SU(N) \right) \right) \ . \tag{17}$$

Here by "$\sim$" we denote homotopic equivalence and $\Sigma(\mathcal{X})$ is a suspension of the manifold $\mathcal{X}$ (see, e.g., Ref. [11] for definitions and notations). The generalized Freidental theorem ([11], p.79) states that $\pi_{q+1}\left(\Sigma\left(\mathcal{X}\right)\right) = \pi_q\left(\mathcal{X}\right)$ for $q \leq 2n-2$, provided $\pi_i(\mathcal{X}) = 0$ for $i < n$. In particular, Eq. (17) implies that $\pi_1(\mathcal{Q}) = \pi_2(\mathcal{Q}) = \pi_3(\mathcal{Q}) = \pi_4(\mathcal{Q}) = 0$ and $\pi_5(\mathcal{Q}) = \mathbb{Z}$.

Therefore, in the case of $SU(N_c)$ SQCD with $N_f = N_c$, the Wess–Zumino term can be constructed as a five-dimensional integral, in similarity to the QCD case. Another outcome of our calculation is that the triviality of the groups $\pi_2(\mathcal{Q})$ and $\pi_3(\mathcal{Q})$ implies that neither vortices nor topological solitons exist in this theory.

The explicit expression generalizing Eq. (16) is

$$\Gamma_{SQCD} = \frac{-1}{12\pi^2\Lambda^{4N}} \,\mathrm{Im} \int_{\Sigma_5} d\Omega \ \det M \cdot \epsilon^{\mu\nu\lambda\rho\sigma} \partial_\mu B \, \partial_\nu \tilde{B} \times \mathrm{Tr}\left(M^{-1}\partial_\lambda M M^{-1}\partial_\rho M M^{-1}\partial_\sigma M \right) \ . \tag{18}$$

It is straightforward to check that Eq. (18) indeed reproduces Eq. (16) at $N = 2$.

It is shown in the Appendix B that $\Gamma_{SQCD} = 2\pi$ when Σ_5 is a generator of $\pi_5(\mathcal{Q})$ and consequently the Wess–Zumino term (18) is unambiguous in quantum theory. At first sight Γ_{SQCD} appears singular at the points where $\det M = 0$. However, this is not the case. Indeed, due to the constraint (10) one has $B, \tilde{B} \neq 0$ at these points, and the integrand in the r.h.s. of Eq. (18) can be rewritten in the following way,

$$\frac{-1}{2}\epsilon^{\mu\nu\lambda\rho\sigma}\partial_\mu \left(\frac{\partial_\nu \tilde{B}}{\tilde{B}} \,\mathrm{Tr}\left(\partial_\lambda M M^{-1} \det M \times \partial_\rho M \partial_\sigma \left(M^{-1} \det M \right) \right) \right) .$$

This expression is explicitly non-singular when $\det M = 0$ because $M^{-1}\det M$ can be defined there by making use of the identity $M^{-1}\det M = \tilde{M}^T$, where $\tilde{M}$ is a matrix composed of the minors of the matrix M.

Let us check that the Wess–Zumino term (18) properly reproduces the induced baryonic charge. Contrary to the QCD case, one can straightforwardly obtain the contribution of the Wess–Zumino term to the baryonic current by making use of the conventional Noether procedure,

$$j_B^\mu = \frac{N}{48\pi^2\Lambda^{4N}} \det M^2 \cdot \epsilon^{\mu\nu\lambda\rho} \times \mathrm{Tr}\left(\partial_\nu M M^{-1} \partial_\lambda M M^{-1} \partial_\rho M M^{-1} \right) + h.c. \tag{19}$$

Then, substituting the vacuum expectation value of the meson fields (14) one obtains the same value of the induced baryonic charge as in the microscopic theory, eq. (13).

To summarize, the above analysis shows that Eq. (18) is a well-defined expression for the Wess–Zumino term in the $SU(N)$ SQCD with quantum modified moduli space and that induced charge matching conditions are satisfied in this theory when the term (18) is taken into account.

IV. SOFTLY BROKEN SQCD

Finally, let us discuss induced charge matching in the model with the soft supersymmetry breaking mass term

$$V_{soft} = \mu_Q^2 (|Q|^2 + |\tilde{Q}|^2) \tag{20}$$

added to the potential of the microscopic theory. It has been suggested in Ref. [12] that, at least at $\mu_Q \ll \Lambda$, the effective theory is described both by the constraint (10) and by the the soft terms

$$V_{eff} = \frac{\mu_B^2}{\Lambda^{2N-2}}(|B|^2 + |\tilde{B}|^2) + \frac{\mu_M^2}{\Lambda^2}|M|^2 \ . \tag{21}$$

Then, up to flavor rotations, there are two candidates for global minima of this potential. In the first one the baryon symmetry is unbroken, $B = \tilde{B} = 0$, $M^a_{\tilde{b}} = \Lambda^2 \cdot \mathbf{1}$, while in the second one the flavor symmetries are unbroken, $B = -\tilde{B} = \Lambda^N$, $M^a_{\tilde{b}} = 0$. Which of these points is the global vacuum depends on the ratio $\frac{\mu_B^2}{\mu_M^2}$ that has not been calculated. Indeed, the baryon number violating stationary point is always a stable minimum, while the stationary point with $B = \tilde{B} = 0$ is unstable if $\frac{\mu_B^2}{\mu_M^2} < 1$ and is a global vacuum if $\frac{\mu_B^2}{\mu_M^2} > \frac{N}{2}$.

Let us now introduce the space-dependent mass term (12). The induced baryonic charge as calculated in the microscopic theory is the same as in the supersymmetric case. In the low-energy theory there are again two candidates for the global minimum of the effective potential. The first one corresponds to the stationary point with unbroken baryon symmetry and is the same as in the supersymmetric case (see Eq. (14)). In the second stationary point the baryon symmetry is broken, $B = -\tilde{B} = \Lambda^N$, $M^a_{\tilde{b}} = 0$. The baryonic current induced in the first candidate vacuum is given by Eq. (19) and properly reproduces the induced charge as calculated in the microscopic theory. In the second extremum the baryon symmetry is spontaneously broken and the baryon charge matching condition cannot be straightforwardly applied. Consequently, the matching condition for the baryonic charge does not enable one to conclude which of the two extrema is the global vacuum.

However, for some special choices of the background scalar field (1), additional matching conditions arise. Namely, let us take the matrix $U^a_{\tilde{b}}(\mathbf{x})$ in the following form,

$$U(\mathbf{x}) = \begin{pmatrix} \tilde{U}(\mathbf{x}) & 0 \\ 0 & \mathbf{1} \end{pmatrix} ,$$

where $\tilde{U}(\mathbf{x})$ is $N_0 \times N_0$ unitary matrix. In analogy to the QCD case, background fields of this form respect two independent symmetries acting non-trivially on the first N_0 flavors. The first one is the baryon symmetry. The second one is a vectorial subgroup of the original $SU(N_f)_L \times SU(N_f)_R$ flavor group; its generator is

$$T^f = \mathrm{diag}\left(1, \ldots, 1, -\frac{N_0}{N_f - N_0}, \ldots, -\frac{N_0}{N_f - N_0}\right) .$$

In the microscopic theory both induced charges are equal to $N_c N[\tilde{U}]$.

In the effective theory the second symmetry remains unbroken in both minima. The corresponding flavor current determined by the Wess–Zumino term (18) is

$$j^\mu = \frac{N}{4\pi^2 \Lambda^{4N}} \det M^2 \cdot \epsilon^{\mu\nu\lambda\rho} \partial_\nu B \, \partial_\lambda \tilde{B} \, \mathrm{Tr}\left(T^f \partial_\rho M M^{-1}\right) + h.c. \tag{22}$$

Therefore, this current is zero in both stationary points. Nevertheless, in the minimum with unbroken baryon symmetry (and in the supersymmetric case as well) the charge matching condition corresponding to the additional symmetry is satisfied. The point is that fermionic components of mesons $\psi_M{}^j_{\tilde{i}}$ are charged under this symmetry provided that one of the indices $(\tilde{i}, j)$ is less than or equal to N_0, while another is larger than N_0. In the first candidate vacuum mesons obtain non-zero vacuum expectation values (14) which generate spatially dependent masses for $\psi_M{}^j_{\tilde{i}}$,

$$\mathcal{A} \frac{\partial^2 \det M}{\partial M^j_{\tilde{i}} \partial M^k_{\tilde{l}}} \psi_M{}^j_{\tilde{i}} \psi_M{}^k_{\tilde{l}}$$

through the term $\mathcal{A} \det M$ in the effective superpotential. Then the calculation of the induced charge proceeds in the same way as in the microscopic theory and one can straightforwardly check that the resulting charge is again $N_c N[\tilde{U}]$.

On the contrary, in the minimum with broken baryon symmetry one has $M^a_{\tilde{b}}(\mathbf{x}) = 0$ and the relevant fermions remain massless. Consequently, charge matching conditions are not satisfied in this minimum, that strongly suggests that the baryon symmetry is unbroken in the global vacuum [4] and that $\frac{\mu_B^2}{\mu_M^2} > \frac{N}{2}$.

[4]It is worth noting, however, that in the whole range $\frac{\mu_B^2}{\mu_M^2} < \frac{N}{2}$ there is a *local* vacuum in which matching conditions are satisfied. For $\frac{\mu_B^2}{\mu_M^2} > 1$ this is the vacuum with unbroken baryon symmetry and for $\frac{\mu_B^2}{\mu_M^2} < 1$ there appears an additional local minimum where both baryon and flavor symmetries are broken.

V. CONCLUSIONS

We have considered matching of induced charges in $SU(N)$ SQCD with quantum modified moduli space. We have found that matching conditions are satisfied provided an additional term similar to the Wess–Zumino term in the non-supersymmetric QCD is added. Our calculation has shown that, contrary to the QCD case, the third homotopy group $\pi_3(\mathcal{Q})$ of the vacuum space is trivial. Consequently, no topological solitons exist in SQCD. Another consequence of this fact is that in the supersymmetric case, the baryonic current carried by spatially inhomogeneous meson fields is not topological and can be obtained by making use of the conventional Noether procedure. The second homotopy group $\pi_2(\mathcal{Q})$ was found to be also trivial, so there are no global vortices in this theory either.

We have studied the case of softly broken supersymmetry as well. In similarity to the models considered in Refs. [3,4], induced charge matching conditions provide non-trivial information about the low-energy effective theory in this case. Namely, they strongly disfavor the existence of the global baryon number violating vacuum. As a consequence, we have obtained the lower bound on the ratio of the soft baryonic and mesonic masses.

VI. ACKNOWLEDGMENTS

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APPENDIX A. TOPOLOGY OF THE QUANTUM MODULI SPACE

In this Appendix we present a proof of the homotopic equivalence given by Eq. (17). Let us first recall that the suspension $\Sigma(\mathcal{X})$ of the manifold $\mathcal{X}$ is the cylinder $\mathcal{X} \times [0, 1]$ where all points on the lower base $\mathcal{X} \times 0$ are identified and all points on the upper base $\mathcal{X} \times 1$ are identified as well. As an example, the suspension of the d-dimensional sphere S^d is the $(d+1)$-dimensional sphere $S^{(d+1)}$. The latter observation is the basis of the generalized Freidental theorem ("theorem about suspension"), which we refer to in the text.

In order to prove the homotopic equivalence (17), let us consider new variables B_1 and B_2 instead of $B, \tilde{B}$ such that the quantum moduli space is determined by the following equation,

$$\det M = \Lambda^{2N} - B_1^2 - B_2^2.$$

Let us first consider the manifold $\mathcal{Q}_1$ defined by the constraint

$$\det M = \Lambda^{2N} - B_1^2, \tag{23}$$

and show that $\mathcal{Q}_1$ is homotopically equivalent to $\Sigma(SL(N,\mathbb{C}))$. Indeed, the surface defined by Eq. (23) at a fixed value of $B_1 \neq \pm\Lambda^N$ is topologically equivalent to $SL(N,\mathbb{C})$. At $B_1 = \pm\Lambda^N$, Eq. (23) defines the surfaces homotopically equivalent to a point. Consequently, Eq. (23) indeed defines the manifold homotopically equivalent to $\Sigma(SL(N,\mathbb{C}))$ if one restricts B_1 to belong to the interval $[-\Lambda^N, \Lambda^N]$. Finally, let us construct a deformation retract of $\mathcal{Q}_1$ onto its part determined by $B_1 \in [-\Lambda^N, \Lambda^N]$.

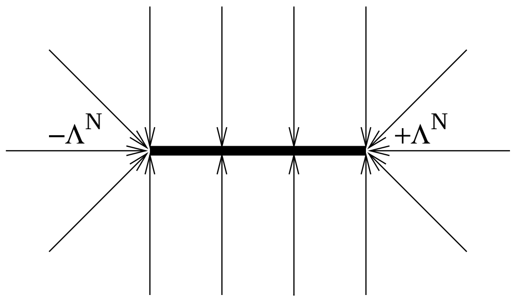

FIG. 1. Trajectories of the deformation $\mathcal{Q}_1 \to \Sigma(SL(N,\mathbb{C}))$ in the complex plane B_1.

Let us take a point $(M_{\tilde{b}}^a(0), B_1(0)) \in \mathcal{Q}_1$ with $B_1(0) \neq \pm\Lambda^N$. Let the coordinate B_1 move in the complex plane as shown in Fig. 1 and define the matrix $M_{\tilde{b}}^a(t)$ at the moment t as follows,

$$M(t) = TM(0) \ ,$$

where

$$T = \begin{pmatrix} \frac{\Lambda^{2N} - B_1(t)^2}{\Lambda^{2N} - B_1(0)^2} & 0 \\ 0 & \mathbf{1} \end{pmatrix}$$

and $\mathbf{1}$ is the $(N-1)\times(N-1)$ unit matrix. The existence of such a deformation implies that $\mathcal{Q}_1 \sim \Sigma(SL(N,\mathbb{C}))$.

One can straightforwardly generalize these arguments and show that $\mathcal{Q} \sim \Sigma(\mathcal{Q}_1)$. The last remark to be made for proving the relation (17) is that the $SU(N)$ group is homotopically equivalent to the $SL(N,\mathbb{C})$ group.

APPENDIX B. UNAMBIGUITY OF THE WESS–ZUMINO TERM IN QUANTUM THEORY

Γ_{SQCD} is unambiguous in quantum theory iff it is equal to $2\pi n$, $n \in \mathbb{Z}$, when Σ_5 is the generator of $\pi_5(\mathcal{Q})$. The theorem about suspension implies the following construction of this generator. Let us consider a sphere S^5 with unit radius,

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = 1 \ , \quad x_i \in \mathbb{R} \ .$$

The generator of $\pi_5(\mathcal{Q})$ is the map $S^5 \to \mathcal{Q}$ that in the notations of Appendix A can be written as follows

$$B_1 = \Lambda^N x_5 \ ,$$

$$B_2 = \Lambda^N x_6 \ ,$$

$$M_{\tilde{b}}^a = \Lambda^2 (1 - x_5^2 - x_6^2)^{\frac{1}{N}} U_{\tilde{b}}^a(x_1, x_2, x_3, x_4) \ ,$$

where $U_{\tilde{b}}^a(x_1, x_2, x_3, x_4)$ defines a generator of $\pi_3(SU(N))$. Taking into account the expression (5) for the topological number of the map $S^3 \to SU(N)$ and recalling that $dB \wedge d\tilde{B} = 2i dB_1 \wedge dB_2$ one obtains

$$\Gamma_{SQCD} = 4 \int_{x_5^2 + x_6^2 \leq 1} dx_5 dx_6 (1 - x_5^2 - x_6^2) = 2\pi$$

when Σ_5 is the generator of $\pi_5(\mathcal{Q})$. Consequently, the Wess–Zumino term (18) is indeed unambiguous in quantum theory.

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